TIPE: Low-frequency acoustic filtering using a Helmholtz resonator

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TIPE

2024

Plan de la présentation

- ***Droning*** is an ***undesirable*** sound whose frequency lies between 150 Hz and 300 Hz.
- HR: Helmholtz resonator
- HRS: attenuation system composed of a Helmholtz resonator
- DHRS: attenuation system composed of a coupled Helmholtz resonator

Research Question

How can we, based on the theory of the Helmholtz resonator, design a quiet, efficient, and tunable system?

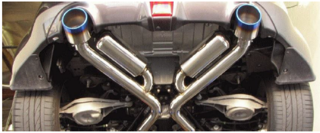

Figure: Left: Car exhaust, HRS. Right: Helmholtz resonator

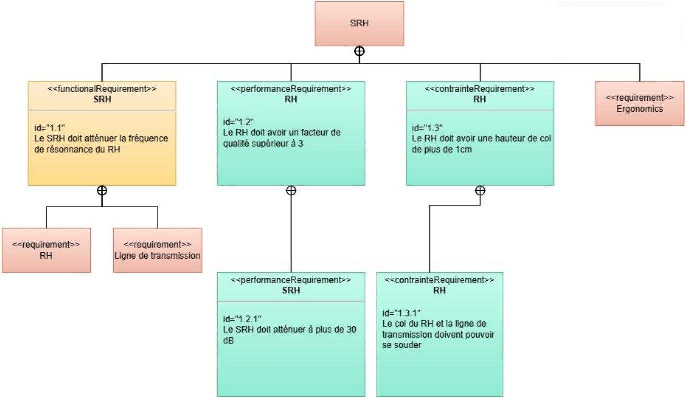

Figure: Requirement diagram

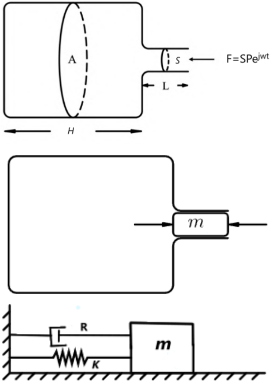

Figure: Modeling

Notations

- V: Cavity volume
- ρ: Air density
- K: Stiffness constant of air
  $K = \frac{S^2 \rho c^2}{V}$
- m: Mass of air contained in the neck $\quad m = \rho S L$
- R: Radiative loss resistance

Natural Frequency

$$f_0 = \frac{c}{2\pi}\sqrt{\frac{S}{VL}} \tag{1}$$

Acoustic Impedance

$$Z = \frac{P}{U} = \frac{R}{S^2} + j\frac{\rho L}{S\omega}(\omega^2 - \omega_0^2) \tag{2}$$

Quality Factor

$$Q = 2\pi\sqrt{V\left(\frac{L}{S}\right)^3} \tag{3}$$

Protocol

- Design PVC HRs with a high quality factor Q
- Place the microphone inside the HR at f_0

Objective

- Measure the variation of acoustic pressure inside the HR to identify f_0

Figure: PVC HR

V(cm^3)	L(cm)	r(mm)	f_0(Hz)	Q
706±6	1.61±0.05	10.50±0.03	300±5	40±1

- **Microphone:** *Behringer ECM8000* **Sound card:** *Audiobox Go*

SPL

$$SPL = L_p + 20\log_{10}\left(\frac{p}{p_0}\right)$$

$$\begin{aligned} SPL &: SoundPressureLevelindecibels(dB), \\ L_p &: ReferencesoundpressurelevelindB, \\ p &: Effectiveacousticpressureofthesoundinpascals, \\ p_0 & \end{aligned}$$

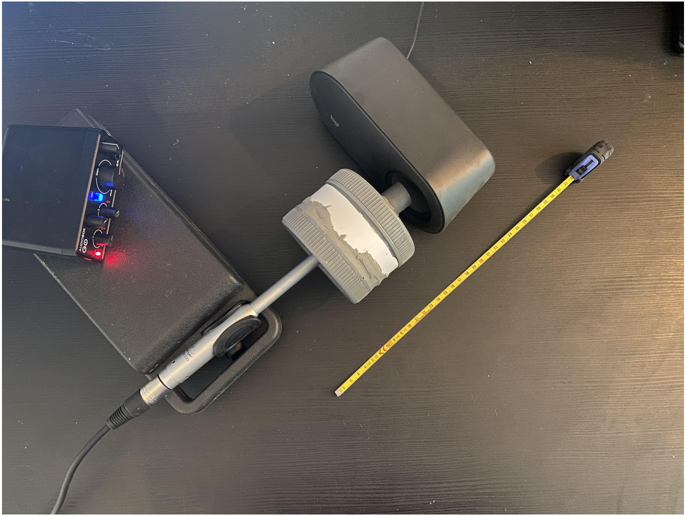

Figure: HR Experiment

I - Study of the Resonator

Experimental Result

$$Q_{\text{exp}} = \frac{113}{316 - 295} = 5 \pm 1$$

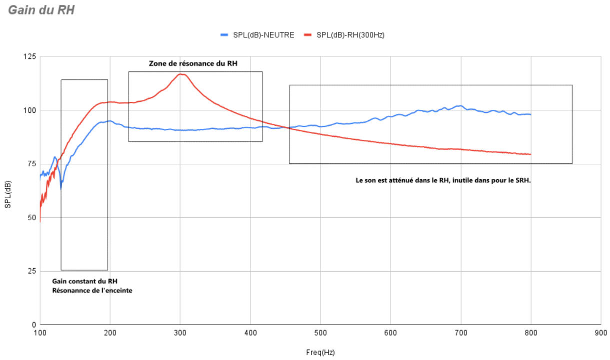

Figure: Experimental HR Gain

f_0(Hz)	221.28	191.64	171.4	156.48	144.86	135.50	127.76
L(cm)	3	4	5	6	7	8	9

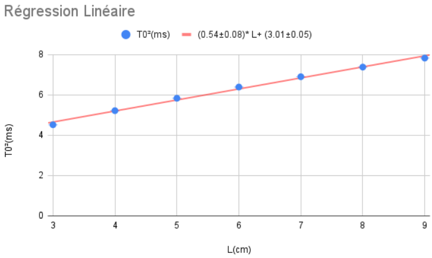

Figure: Linear Regression

Or $T_0^2 = \left(\frac{2\pi}{c}\right)^2 \frac{VL}{S}$

Estimate: $\frac{2\pi}{c}^2 \frac{V}{S} = 0.69 s/m$

Normalized deviation: $z = \frac{0.54 - 0.69}{0.51} = 1.875$

I - Study of the Resonator

Bode Diagram

$$H(p) = \frac{\frac{pS}{\rho L \omega_0^2}}{1 + \frac{R}{\rho L S \omega_0^2} p + \frac{1}{\omega_0^2} p^2}$$

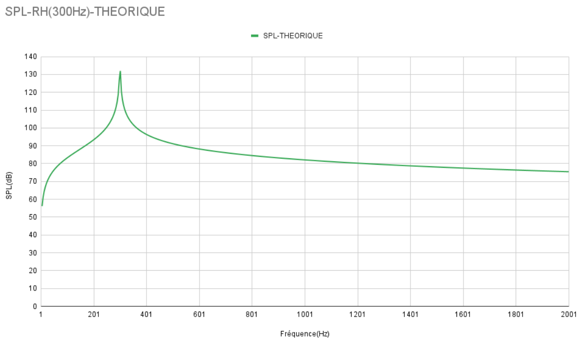

Figure: HR Gain Plot with Python

Linearized NavierStokes Equation in Velocity

$$\rho\vec{a} = -\vec{\nabla}P + \rho\vec{g} - \eta\vec{\nabla}\vec{v}, \quad \vec{a} = \frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v}$$

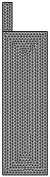

Figure: Mesh

Simulation Result

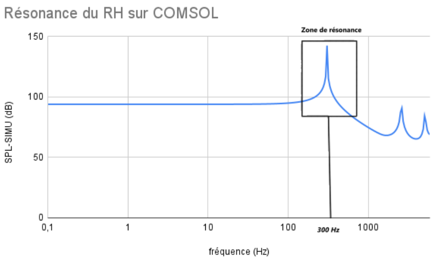

Figure: COMSOL Resonance HR

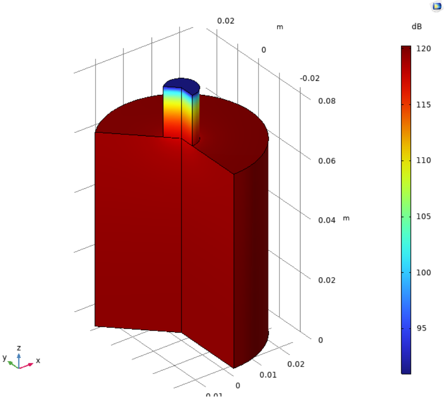

Figure: HR at 300 Hz

- The three **resonances** coincide, enabling resonance prediction and HR sizing.
- Q_{exp} meets the requirements.
- R_{visc} is not negligible, making it difficult to size the quality factor Q of the HR.

Q_{exp}	Q_{simu}	Q_{theo}
4 ± 1	45 ± 1	40 ± 1

We can estimate $R_{visc} = \frac{\omega_0 m}{Q_{exp}}$

$R_{visc} = (0.0022 \pm 0.0005)\,\mathrm{kg/s}$

I - Study of the Resonator
Comparisons

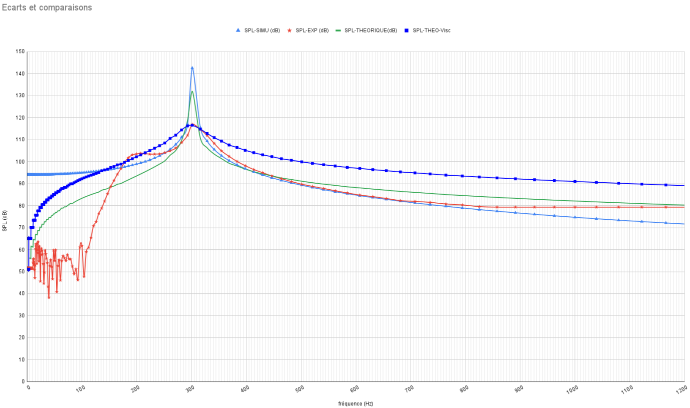

Figure: Comparison of HR results

Transmission Loss

Definition:

$$T_L = 10 \log_{10} \frac{W_i}{W_t}$$

- W_i: energy of the incident acoustic wave
- W_t: energy of the transmitted acoustic wave

Moreover, noting I as the sound intensity:

$$W = IS = \frac{P^2}{2\rho c}$$

Transmission Loss

$$TL = 20 \cdot \log_{10} \left(\frac{P_i}{P_t} \right)$$

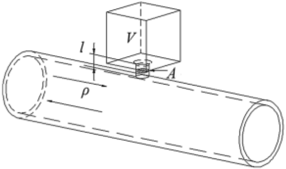

Figure: HRS model

- According to **[10]** and the four-pole method:

TL

$$TL = 20\log\left(\frac{1}{2}\left|2+\frac{\rho_{\text{air}}\,c}{S_d Z}\right|\right)$$

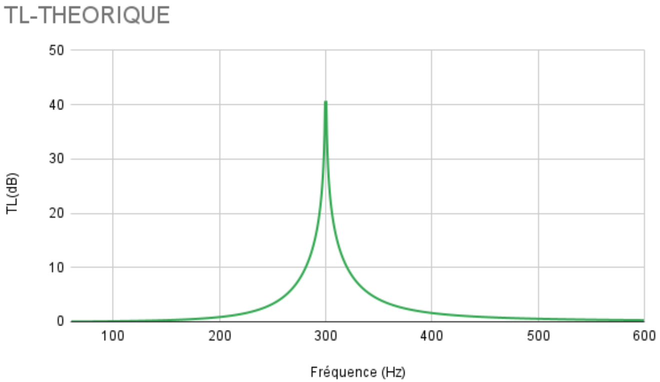

Figure: Theoretical TL

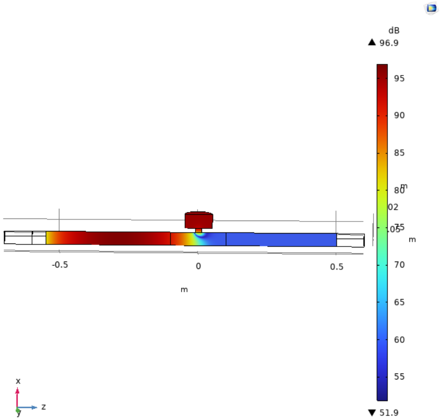

Figure: HRS at 300 Hz

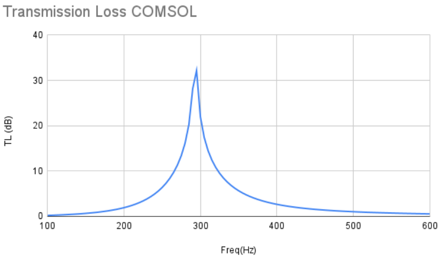

Figure: COMSOL TL simulation

- Exhaust pipes are modeled using PVC tubes. Kundt tubes (0.56 m) resonating at 300 Hz.

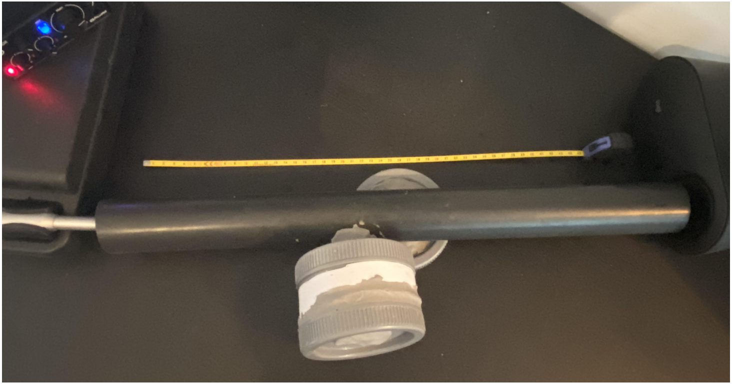

Figure: HRS experiment

II - Study of the Attenuation System

Experimental Result of HRS in SPL

- We then obtain:

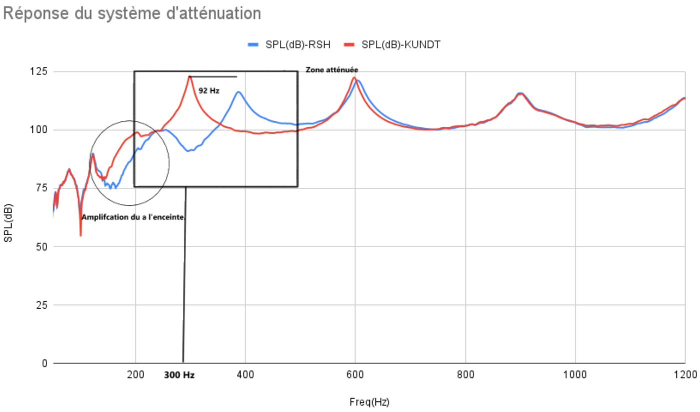

Figure: SPL HRS

Transmission Loss

$$TL = SPL_{Kundt} - SPL_{HRS}$$

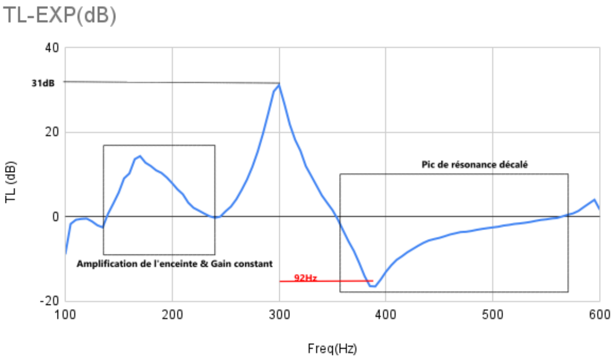

Figure: HRS TL-EXP

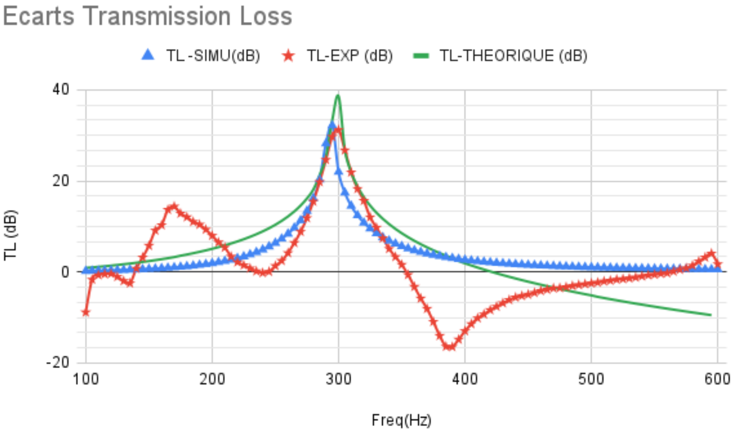

Figure: HRS deviations

- The HRS meets the attenuation requirement.
- Anti-resonance from the coupling of Kundt tube and HR may reintroduce droning.

III - Modulation of the Natural Frequency of the HRS

Figure: Experimental DHRS

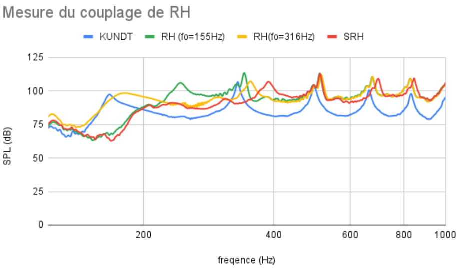

Figure: Experimental DHRS curve

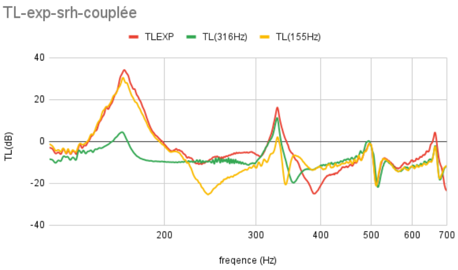

Figure: Experimental DHRS TL

- Using impedance (5.4) and **[10]**, the TL of the DHRS can be plotted.
- Z_{eq} is used for resonators in parallel.

Transmission Loss

$$TL = 20\log\left(\frac{1}{2}\left|2 + \frac{\rho_{\text{air}}\, c}{S_d Z_{eq}}\right|\right)$$

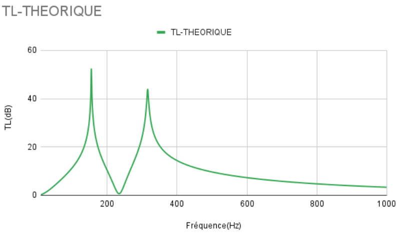

Figure: Theoretical DHRS TL

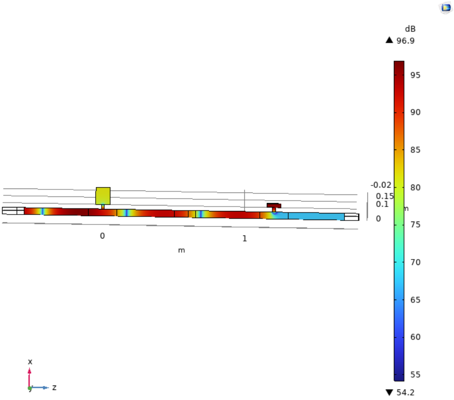

Figure: Small HR resonance (316 Hz)

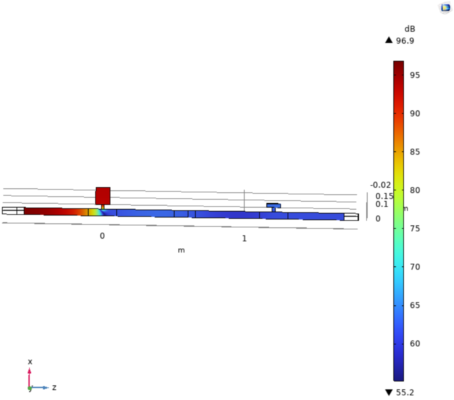

Figure: Large HR resonance (155 Hz)

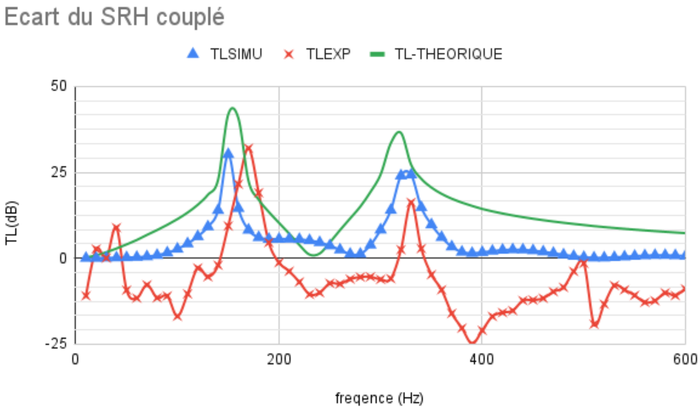

Figure: Comparisons

Conclusion

- I highlighted the resonance properties of the HR and empirically determined an approximation of R_{visc} to size the quality factor Q.
- I designed an HRS that meets the requirements, achieving attenuation greater than 30 dB.
- Through coupling, it is possible to tune the resonances of the HRS in order to counter the anti-resonance phenomenon.
- Considering all these parameters, the HRS or DHRS can be integrated into an exhaust line to mitigate one or several disturbances.

Conclusion

Figure: HRS on a SUBARU

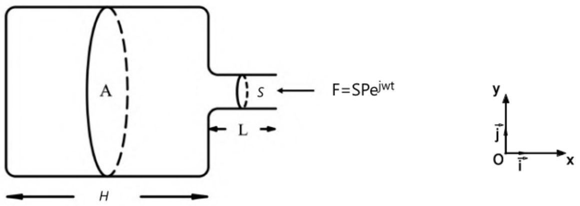

Figure: Resonator diagram

Assumptions

- The model remains valid if $\lambda \gg L$, $\lambda \gg V^{\frac{1}{3}}$, and $\lambda \gg S^{\frac{1}{2}}$
- For our resonator, the effective neck length is defined as $L' = L + 1.7S$

Under the effect of an external perturbation $F = SPe^{jwt}$, the differential equation governing the evolution of ξ, the amplitude of a small displacement of the air column in the neck, is:

Differential equation for ξ displacement in the neck

$$m\frac{d\xi}{dt^2} + R\frac{d\xi}{dt} + K\xi = SPe^{jwt} \tag{4}$$

The resulting mechanical impedance is given by:

$$Z_m = R + j(\omega m - \frac{K}{m}) \tag{5}$$

Resonance occurs when the reactance tends toward zero.

Natural Angular Frequency

$$\omega_0 = c\sqrt{\frac{S}{LV}} \tag{6}$$

The radiative resistance according to **[9]**, with k as the wave vector, is expressed as:

$$R = \frac{\rho_0 c k^2 S^2}{2\pi} \tag{7}$$

Thus, the quality factor at resonance is expressed as:

$$Q = \frac{\omega_0 m}{R} \tag{8}$$

$$Q = c\sqrt{\frac{S}{LV}} \cdot \rho_0 SL \cdot \frac{2\pi}{\rho_0 c \frac{\omega_0^2}{c^2} S^2} \tag{9}$$

$$Q = 2\pi\sqrt{\frac{S}{LV}} \frac{L^2 V}{S^2} \tag{10}$$

Appendix

Figure 1:
https://motordyneengineering.com/product/shockwave-e370z/
https://patrimoine.auvergnerhonealpes.fr/illustration/IVR82_20134203841NUCA

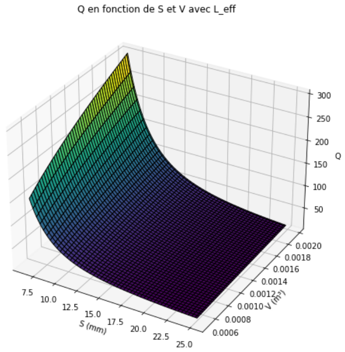

Figure: Evolution of Q

When the piston is displaced by ξ, the cavity volume changes by $\nabla V = -S\xi$. This results in a condensation: $\frac{\nabla\rho}{\rho} = \frac{\nabla V}{V} = \frac{S\xi}{V}$. The pressure then increases (within the acoustic approximation):

$$p = \frac{\rho_0 c^2 S\xi}{V} \tag{11}$$

The force f required to maintain the displacement is $f = Sp = K\xi$. The stiffness is therefore:

$$K = \frac{\rho_0 c^2 S^2}{V} \tag{12}$$

Similarly, an RLC series circuit can be introduced as an analogy of a first-order damped oscillator.

- Acoustic capacitance:

$$C_a = \frac{S^2}{K}$$

- Acoustic mass:

$$M_a = \frac{m}{S^2}$$

- Acoustic resistance:

$$R_a = \frac{R}{S^2}$$

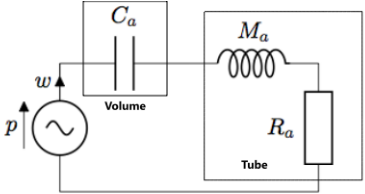

Figure: RLC circuit

- Similarly, noting x the displacement of the air mass in the neck, and $v = \frac{dx}{dt}$ such that $U = Sv$ with P as the excitation pressure at the neck inlet:

$$Z = \frac{P}{U} = R_a + j(\omega M_a - \frac{1}{C_a}\omega) = R_a + j(\omega \frac{M_m}{S^2} - \frac{K_m}{\omega S^2}) \tag{13}$$

$$Z = R_a + j(\omega \frac{\rho SL}{S^2} - \frac{K_m}{\omega S^2}) = R_a + j\frac{\rho L}{S\omega}\left(\omega^2 - \omega_0^2\right) \tag{14}$$

Appendix

Appendix II: REW Audio Acquisition

- Example of acquisition for an HR (366 Hz)
- Example of the influence of cavity volume on Q

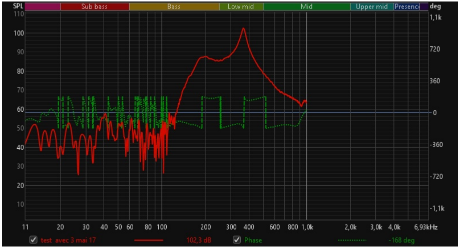

Figure: HR (366 Hz)

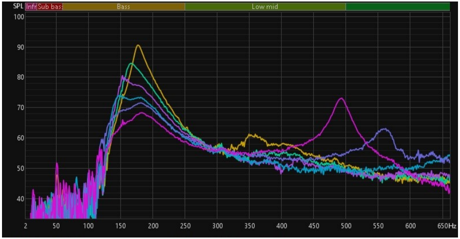

Figure: Variation of Q

- Example of the harmonic decomposition of the audio signal of an HR (366 Hz). The noise floor of 7.31% estimates the measurement uncertainty at 1 dB.

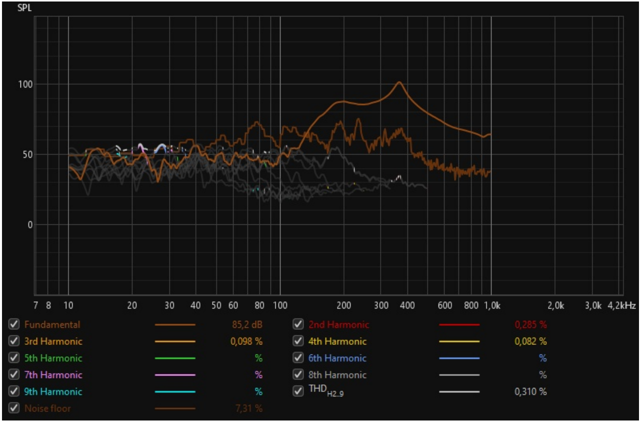

Figure: Spectral decomposition

Appendix

Appendix *III*: COMSOL

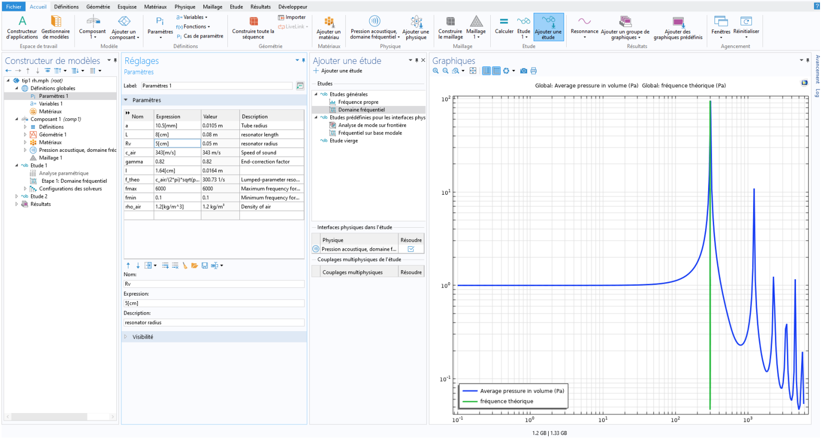

Figure: COMSOL HR

- Function code used to superimpose all obtained results

```
def trouver_valeurs_correspondantes(f1, f2, V2):
    valeurs_correspondantes = []

    f1_filtered = [freq for freq in f1 if freq is not None]
    f2_filtered = [freq for freq in f2 if freq is not None]
    V2_filtered = [valeur for valeur in V2 if valeur is not None]

    for freq1 in f1_filtered:
        indice_plus_proche = min(range(len(f2_filtered)), key=lambda i: abs(f2_filtered[i] - freq1))
        valeurs_correspondantes.append(V2_filtered[indice_plus_proche])

    return valeurs_correspondantes
```

- Code to plot the theoretical HR

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

# Resonator parameters
V_cm3 = 706.9
l_cm = 1.6
r_mm = 10.5
f0_Hz = 300

# Unit conversions
V_m3 = V_cm3 * 1e-6
l_m = l_cm * 0.01
r_m = r_mm * 0.001

# Additional parameters
rho_air = 1.2
Ra = 0.0001 / (np.pi * r_m**2)**2

# Frequency range
frequency_values = np.linspace(1, 2000, 500)
omega_values = 2 * np.pi * frequency_values
```

```
impedance = Ra + (1j/omega_values*(np.pi * r_m**2)) * \
(rho_air*1e-6 * l_m) / V_m3 * (omega_values**2 - (2 * np.pi * f0_Hz)**2)

gain_values = 20 * np.log10(np.abs(1/impedance))+70

df = pd.DataFrame({'Frequency (Hz)': frequency_values,
                   'Gain (dB)': gain_values})

gain_max = gain_values.max()
index_max = gain_values.argmax()
f_max = frequency_values[index_max]

df.to_csv('resonator_results.csv', index=False)

plt.figure(figsize=(10, 6))
plt.semilogx(frequency_values, gain_values)
plt.title('Bode Diagram  Gain')
plt.xlabel('Frequency (Hz)')
plt.ylabel('Gain (dB)')
plt.grid(True)
plt.show()
```